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THE CHROMATIC NUMBER OF THE FUNCTION

The construction of the mathematical objects of the group structure on the set which is under the study and the use of the properties of this structure is one of the effective methods of the study. The concept of homomorphism is one of the basic concepts of group theory. This concept is very useful under the study of the properties of the groups. Homomorphism is the mapping from one group to another which preserves the group operation. An analogue of the concept of homomorphism in the case when an arbitrary everywhere defined mapping $f : X^n \to X$ is given instead of the group operation has been constructed by the authors. The case when $n = 2$ and $X \subset R$ have been studied in details in the article. The concept of the chromatic number of this mapping and the examples of its calculation have been given. The examples of the chromatic numbers of the certain groups have been given with the necessary explanations. The concept of the chromatic number of the real numerical function has been introduced. It has been shown that this concept is closely linked to the concept of V.L. Rvachev R - function. It has been shown, using the known results, that the functions with the infinite chromatic numbers exist. The examples of the chromatic numbers for the certain functions have been given with the necessary explanations. The main result of this article is the proof of the fact that the linear function of two real variables $f(x, y) = \alpha x + \beta y + \gamma, \alpha\beta \neq 0$ has no finite chromatic number. The similar result has been proved for the function $g(x, y) = x^2 - y^2$ of two real variables. Thus, the set R can not be colored into the finite number of the colors in such a way that the color of the value of the function $\alpha x + \beta y + \gamma$, where $\alpha\beta \neq 0$ is uniquely determined by the colors of its arguments. The same fact is true for the function $x^2 - y^2$ and $ax_1x_2...x_n + b$, where $n > 1, ab \neq 0$. The obtained result can be formulated in terms of R - function as follows:

the functions $f(x, y)$ and $g(x, y)$ (as well as the function $ax_1x_2...x_n + b$ under $n > 1, ab \neq 0$) can not be R - function at any choice of the accompanying functions of multiple-valued logic.

Thus, the concepts of the chromatic class of the function and the chromatic number of the function have been introduced in the given article. The relation between the obtained concepts and group theory has been found. It has been demonstrated that the concept of the chromatic number of the function on the certain set is closely linked to the concept of V.L. Rvachev R - function. It has been pointed out that the fact that the chromatic numbers and the chromatic classes coincide for the isomorphic groups can be used under proving of the nonisomorphy of the groups.

Keywords: chromatic number of the function, R - function, homomorphism.

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ХРОМАТИЧЕСКОЕ ЧИСЛО ФУНКЦИИ

Построение на исследуемом множестве математических объектов групповой структуры и использование ее свойств является одним из эффективных методов исследования. Одним из центральных понятий теории групп является понятие гомоморфизма, которое оказывается очень полезным при изучении свойств групп. Гомоморфизм – это отображение из одной группы в другую, которое сохраняет групповую операцию. В данной статье авторами построен аналог понятия гомоморфизма на случай, когда вместо групповой операции задается произвольное, всюду определенное отображение $f : X^n \to X$. В статье подробно рассматривается случай, когда $n = 2$ и $X \subset R$. Дается определение хроматического числа этого отображения и приводятся примеры его вычисления. Приведены примеры хроматических чисел некоторых групп с необходимыми пояснениями. Введено

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понятие хроматического числа вещественной числовой функции и показано, что это понятие тесно связано с понятием R - функции В.Л. Рвачева. Опираясь на известные ранее результаты, показано, что существуют числовые функции с бесконечными хроматическими числами. В качестве примеров приведены хроматические числа некоторых функций, даны пояснения полученных результатов. Основным результатом этой статьи является доказательство того факта, что линейная функция двух действительных переменных $f(x,y)=\alpha x+\beta y+\gamma,\ \alpha\beta\neq 0$ не имеет конечного хроматического числа. Аналогичный результат доказан для функции $g(x,y)=x^2-y^2$ двух действительных переменных. Таким образом, множество R нельзя раскрасить в конечное число цветов так, чтобы цвет значения функции $\alpha x+\beta y+\gamma$, где $\alpha\beta\neq 0$ однозначно определялся цветом ее аргументов. То же касается функций x^2-y^2 и $ax_1x_2...x_n+b$, где $n>1, ab\neq 0$. В терминах R - функций полученный результат можно сформулировать следующим образом:

функции $f(x,y)$ и $g(x,y)$ (как и функция $ax_1x_2...x_n+b$ при $n>1, ab\neq 0$) не могут быть R - функциями ни при каком выборе сопровождающих функций многозначной логики.

Таким образом, в данной статье введены понятия хроматического класса и хроматического числа функции. Найдена связь между полученными понятиями и теорией групп. Продемонстрировано, что понятие хроматического числа функции на некотором множестве тесно связано с понятием R - функции В.Л. Рвачева. Отмечено, что для доказательства неизоморфности групп можно использовать тот факт, что для изоморфных групп хроматические числа и хроматические классы, к которым они относятся, совпадают.

*Ключевые слова**:** хроматическое число функции, R - функция, гомоморфизм.*

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ХРОМАТИЧНЕ ЧИСЛО ФУНКЦІЇ

Побудова на досліджуваній множині математичних об'єктів групової структури і використання її властивостей є одним з ефективних методів дослідження. Одним з центральних понять теорії груп є поняття гомоморфізму, яке виявляється дуже корисним при вивченні властивостей груп. Гомоморфізм - це відображення з однієї групи в іншу, яке зберігає групову операцію. У даній статті авторами побудований аналог поняття гомоморфізму на випадок, коли замість групової операції задається довільне, усюди визначене відображення $f: X^n \to X$. У статті докладно розглядається випадок, коли $n=2$ і $X\subset R$. Дається визначення хроматичного числа цього відображення і наводяться приклади його обчислення. Наведені приклади хроматичних чисел деяких груп з необхідними поясненнями. Введено поняття хроматичного числа дійсної числової функції і показано, що це поняття тісно пов'язане з поняттям R - функції В.Л. Рвачева. Спираючись на відомі раніше результати, показано, що існують числові функції з нескінченними хроматичними числами. Для прикладу наведено хроматичні числа деяких функцій, дані пояснення отриманих результатів. Основним результатом цієї статті є доведення того факту, що лінійна функція двох дійсних змінних $f(x,y)=\alpha x+\beta y+\gamma,\ \alpha\beta\neq 0$ не має скінченого хроматичного числа. Аналогічний результат доведений для функції $g(x,y)=x^2-y^2$ двох дійсних змінних. Таким чином, множину R неможна розфарбувати в скінчене число кольорів так, щоб колір значення функції $\alpha x+\beta y+\gamma$, де $\alpha\beta\neq 0$ однозначно визначався кольором її аргументів. Те ж стосується функцій x^2-y^2 і $ax_1x_2...x_n+b$, де $n>1, ab\neq 0$. У термінах R - функцій отриманий результат можна сформулювати наступним чином:

функції $f(x,y)$ і $g(x,y)$ (як і функція $ax_1x_2...x_n+b$ при $n>1, ab\neq 0$) не можуть бути R - функціями ні при якому виборі супроводжуючих функцій багатозначної логіки. Таким чином, в даній статті введено поняття хроматичного класу і хроматичного числа функції. Знайдено зв'язок між отриманими поняттями і теорією груп. Продемонстровано, що поняття хроматичного числа функції на деякій множині тісно пов'язане з поняттям R - функції В.Л. Рвачева. Відзначено, що для доведення

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неізоморфності груп можна використовувати той факт, що для ізоморфних груп хроматичні числа і хроматичні класи, до яких вони належать, збігаються.
Ключові слова: хроматичне число функції, R - функція, гомоморфізм.

The formulation of the problem

The concept of homomorphism, which is very useful for the studying of the properties of the groups [2], is one of the basic concepts of group theory. The homomorphism is the group operation which preserves the mapping from one group into the other group. An analogue of the concept of the homomorphism in the case when the arbitrary completely defined mapping $f : X^n \to X$ is given instead of the group operation has been proposed by the authors. The case $n = 2$ and $X \subset R$ has been studied thoroughly. The definition of the chromatic number [1] of this mapping and the examples of its evaluation have been given.

The analysis of the recent research and publications

The concept of V.L. Rvachev R - function has been described in [3, 4]. The fact that the function $f(x, y) = xy - 1$ can not be R - function has been proved in the article [5].

The aim of the study

The aim of this article is to prove that the linear function of two real variables $f(x, y) = \alpha x + \beta y + \gamma, \alpha\beta \neq 0$ and the function $g(x, y) = x^2 - y^2$ of two real variables have no finite chromatic number.

The presentation of the main material

Let G - be some group. If there exists such finite group T, which contains $k > 1$ elements, and $f : G \to T$ - surjective homomorphism, then let us say that the group G belongs to the chromatic k - class. The minimal nontrivial group, for which it is possible to construct such homomorphism, is of grate interest. Let us call the minimal number of the chromatic class, to which the given group belongs, the chromatic number of the group G. This name comes from the fact that the elements of the group G are considered to be pained in several colors in such a way that complete inverse image of each element consists of the elements which are alike in color. The homomorphism is given $f : G \to T$.

In other words, the group belongs to the chromatic k - class if it has the normal divisor of index k, and the chromatic number of the group is the minimal index of the nontrivial normal subgroup. Let us consider the **examples.**

1. The chromatic number of the group S_n is equal to 2, because it contains the normal divisor A_n of the index 2. Indeed, if one paints all even substitutions in one color, and the odd substitutions in the other color, then the color of the product of the substitutions can be determined uniquely.

2. The chromatic number of the cyclic group of order p^α is equal to p, where p is the prime number.

3. Group $(Z,+)$ can be mapped onto the group $(Z_n,+)$ for all $n \in N$ naturally, that is this group belongs to the chromatic n - class for all $n \in N \setminus \{1\}$, and its chromatic number is equal to 2.

It is obviously that, the chromatic numbers of the isomorphic groups and the chromatic classes, to which they belong, are the same. One can use this fact to prove the nonisomorphy of the groups.

Let us turn to the generalization of the concepts of the chromatic class and the chromatic number of the group in the case of the arbitrary set. Let the mapping from the

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Cartesian power of the set into this set is given. In this article we restrict ourselves to the case of the mapping with two arguments and $X \subset R$.

Let us consider the arbitrary function $f(x, y)$ over the Cartesian square of some number set X. The range of the function belongs to the set X. The value of the function $f(x, y)$ is considered to be an analog of the result of the group operation on the elements x, y. It stands to reason that it is not necessary to require the associativity of the operation $x * y = f(x, y)$, the existence of the neutral and inverse elements [2].

If such coloring of the set X in the finite number of the colors $k \geq 2$, that the color of the value of the function $f(x, y)$ is determined uniquely by the colors of the values of the arguments x and y exists, then we say that this function belongs to the chromatic k - class. The minimal number of the chromatic class, to which this function belongs, we call the chromatic number of the function $f(x, y)$ on the set X and designate $H(f(x, y), X)$. If the function does not have the finite chromatic number, then we say that it is equal to infinity.

Obviously, if the range of the function $f(x, y)$ is inconsistent with X, and it is the proper subset of X, then $H(f(x, y), X) = 2$. In order to prove this fact we paint the numbers, which belong to the range of the function $f(x, y)$, in one color, and the numbers, which do not belong to the range of this function, in the other color. For example $H(a^2x^2 + b^2y^2 + c, R) = 2$, $H(|f(x, y)| + c, R) = 2$. In these examples $a, b, c \in R$, $f(x, y)$ is arbitrary function on the number plane.

If the set X is the finite one, and if we paint each element in the separate color, then we obtain $H(f(x, y), X) \leq |X|$.

The concept of the chromatic number of the function on some set is closely linked to the concept of V.L. Rvachev R - function [3, 4]. The fact that the function $f(x, y) = xy - 1$ can not be R - function, which was proved in the article [5] before, can be reformulated in the following way: there exist the numerical functions with the infinite chromatic numbers. Among the other things, $H(xy - 1, R) = \infty$.

Let us give the trivial **examples**. We consider the sets R or Z as the set X.

1. The function $f_1(x, y) = x + y$, on the set $Z \times Z$, belongs to all chromatic classes. In order to demonstrate it, we locate the natural number $k \geq 2$ and paint in the same color the numbers, which have equal remainders in division by k. This makes it possible to determine the color of the sum of any couple of integers uniquely. Obviously, the function $f_1(x, y) = x + y$ on the set Z has the chromatic number 2, viz $H(x + y, Z) = 2$. The coloring of the even numbers in one color and the odd numbers in the other color serves as a model of the required coloring in two colors. This result has been formulated above, in the example 3, using the group viewpoint.

2. The chromatic number of the function $f_2(x, y) = xy$ on the set Z is also equal to 2. The coloring of the even numbers in one color and the odd numbers in the other color serves as a model of the required coloring in two colors.

3. The chromatic number of the function $f_2(x, y) = xy$ on the set R is also equal to 2. In order to demonstrate it, let us paint zero in one color, and the rest of the real numbers in the other color. Note that the function $f_2(x, y) = xy$ on the set R can be also associated with the chromatic class 3. For this purpose it is necessary to paint odd numbers, even numbers and zero in different colors.

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The proof of the fact that the chromatic number of the arbitrary linear function and the chromatic number of the function $f(x,y)=x^2-y^2$, on the $R\times R$ are equal to infinity is the main result of this article.

Theorem 1. If $\alpha\beta\neq 0$, then $H(\alpha x+\beta y+\gamma,R)=\infty$.

Proof. Let us suppose that $H(\alpha x+\beta y+\gamma,R)=k$. Then there exists such coloring of the set R in k colors, that one can determine the color of the expression $\alpha x+\beta y+\gamma$ uniquely, if the colors of the arguments are known. Such coloring of the set R determines the equivalence relation on it. Each equivalence class consists of the numbers of the same color. By hypothesis the following relation takes place

$$x_1\sim x_2\wedge y_1\sim y_2\Rightarrow(\alpha x_1+\beta y_1+\gamma)\sim(\alpha x_2+\beta y_2+\gamma). \quad (1)$$

Lemma 1.1. The following implication takes place:

$$x\sim y\Rightarrow \alpha x+t\sim\alpha y+t \quad \forall t\in R. \quad (2)$$

Proof. Let us designate such number that $\beta s+\gamma=t$ by s. It always exists, because $\beta\neq 0$. Then, by reason of (1), we obtain the following

$$x\sim y\wedge s\sim s\Rightarrow(\alpha x+\beta s+\gamma)\sim(\alpha y+\beta s+\gamma)\Leftrightarrow \alpha x+t\sim\alpha y+t.$$

Lemma 1.2. If even one equivalence class, which does not contain 0, involves only one element, then all equivalence classes, which do not contain 0, are one-element classes.

Proof. Let the element $a\neq 0$ be unique in its class. It means, that $z\sim a\Leftrightarrow z=a$. Let us consider the arbitrary element $x\in R, x\neq 0$. We accept the existence of $y\neq x$ and $y\sim x$. Let $t=a-\alpha x$. By applying lemma 1.1, we obtain

$$x\sim y\Rightarrow \alpha x+a-\alpha x=a\sim\alpha y+a-\alpha x=a+\alpha(y-x),$$

viz $x\sim y\Rightarrow a\sim a+\alpha(y-x)$. The last equivalence is possible if only $a=a+\alpha(y-x)$. We obtain $y=x$, taking into the consideration, that $\alpha\neq 0$. It means, that the class, which contains x, is one-element class, which was to be proved.

Let us consider the one-element class, which contains 0. It follows from the lemma 1.2 that, the number of classes is equal to the number of the real numbers, viz it is infinite. This variant is impossible, because the required number of the colors should be finite. Hence, there exists $z\neq 0,\quad z\sim 0$.

Lemma 1.3. If $z\sim 0$, then $n\alpha z\sim 0\ \ \forall n\in Z$.

Proof. Let us use the method of mathematical induction.

For $n=0$ the proposition clearly holds. Let us suppose that it is also true for $n\in Z$, that is $n\alpha z\sim 0\sim z$. We take $t=n\alpha z$ in the forward induction and apply (2). Then

$$z\sim 0\Rightarrow \alpha z+n\alpha z=(n+1)\alpha z\sim n\alpha z\sim 0\Rightarrow(n+1)\alpha z\sim 0.$$

We take $t=(n-1)\alpha z$ in the backward induction. Using the implication (2), we obtain

$$0\sim z\Rightarrow(n-1)\alpha z\sim\alpha z+(n-1)\alpha z=n\alpha z\sim 0\Rightarrow(n-1)\alpha z\sim 0.$$

Lemma 1.4. If $x\sim y$, then $\alpha(x-y)\sim\alpha(y-x)\sim 0$.

Proof. Let us apply lemma 1.1 for $t=-\alpha y$. We obtain

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$$x \sim y \Rightarrow \alpha x - \alpha y = \alpha(x - y) \sim \alpha y - \alpha y = 0 .$$

One can prove the second part of the statement by a similar way.

Let us prove the theorem 1. Put $H(\alpha x + \beta y + \gamma, R) = k$. Let us consider the element z such that $z \not\sim 0$. Let us denote $y = \frac{1}{\alpha^2}\frac{z}{k!}$. The system $\{0, y, 2y, 3y, ..., ky\}$ contains $k+1$ elements, which are painted in k colors. According to Dirichlet principle, at least two elements from this system are painted in the same color. Let it be the elements py and $(p+m)y$, besides $0 < m < k$. According to lemma 1.4

$$\alpha((p+m)y - py) = m\alpha y \sim 0 .$$

Since $0 < m < k$, then $n = \frac{k!}{m}$ is integer. According to lemma 1.3 the number $\alpha n(m\alpha y) = \frac{k!}{m} m\alpha^2 y = \frac{k!}{\alpha^2}\frac{\alpha^2 z}{k!} = z$ is equivalent to 0, but this is in contrast with the selection of the element z . **The theorem is proved**.

Remark. The conclusion of the theorem 1 remains true if one considers the set Q ($\alpha, \beta, \gamma \in Q$) instead of the set R .

Corollary. There are no subgroups of the normal index of the groups $(R,+)$ and $(Q,+)$.

Proof. Let us confine ourselves to the consideration of the set R . The proof for the set Q is similar to the proof for the set R .

We take $\alpha = \beta = 1$, $\gamma = 0$ in the theorem, then $H(x + y, R) = \infty$. Let us assume the contrary. The group $(R,+)$ has the subgroup M (which is normal subgroup because of the commutativity of the group R) such that $R/M \cong P$, where $|P| = k$, $1 < k < \infty$. Let us designate the mapping, which defines the chosen homomorphism, by f .We paint all the elements in each of the complete inverse images of the elements from P in the same color. At such coloring two real numbers are alike in color when and only when their images are the same.

Let us consider two sums $a + b$ and $\tilde{a} + \tilde{b}$, the corresponding elements of which are alike in color. Let us prove that these sums are alike in colors. By virtue of the fact that $f(a+b) = f(a) + f(b) = f(\tilde{a}) + f(\tilde{b}) = f(\tilde{a} + \tilde{b})$. Hence, the color of the sum is defined uniquely by the colors of the addends. It means that the function $x + y$ on R belongs to the chromatic k - class. It is in contrast with the statement $H(x + y, R) = \infty$. **The corollary is proved**.

Let us give an example of the quadratic polynomial, the chromatic number of which is infinite.

Theorem 2. $H(x^2 - y^2, R) = \infty$.

Put $H(x^2 - y^2, R) = k$. The set R is divided into the finite number of the equivalence classes. Each class consists of the numbers, which are painted in the same color. The following relation takes place

$$x_1 \sim x_2 \wedge y_1 \sim y_2 \Rightarrow (x_1^2 - y_1^2) \sim (x_2^2 - y_2^2). \quad (3)$$

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Lemma 2.1. If $x \sim y$, then $x^2 - y^2 \sim 0$.

Proof. One can prove the lemma using the formula (3) for the equivalences $x \sim y \wedge y \sim y$.

Lemma 2.2. If $x \sim 0$, then

$$t^2 \sim t^2 - x^2 \quad \forall t \in R \tag{4}$$

and

$$-t^2 \sim x^2 - t^2 \quad \forall t \in R . \tag{5}$$

Proof. The relation (4) follows from the equivalences $t \sim t \wedge 0 \sim x$ and from the formula (3). The relation (5) follows from the equivalences $0 \sim x \wedge t \sim t$.

Lemma 2.3. If $z \sim 0$, then $mz^2 \sim 0 \ \ \forall m \in Z$.

Proof. Let us apply induction on m. If $m = 0$ the proposition clearly holds. We obtain $z^2 \sim 0 \sim -z^2$, using lemma 2.2 for the relation $z \sim 0$, when $t = 0$. Let us prove, that the statement is valid for all positive integers m. For $m = 1$ the statement is proved. Let it be true for some $m > 0$, that is $mz^2 \sim 0$. We put $t^2 = (m+1)z^2$ and we apply formula (4) for the equivalence $z^2 \sim 0$. We obtain $(m+1)z^2 \sim (m+1)z^2 - z^2 = mz^2 \sim 0$.

Let us prove that the statement is true for all negative integers m. For $m = -1$ the statement is proved.

Let it be true for some $m < 0$, that is $mz^2 \sim 0$.

We put $t^2 = -(m-1)z^2$ and we apply formula (5) for the equivalence $z^2 \sim 0$.

We obtain $-\left(-(m-1)z^2\right) = (m-1)z^2 \sim z^2 - \left(-(m-1)z^2\right) = mz^2 \sim 0$.

Let us return to the proof of the theorem 2. If arbitrary $s > 0$, then there exists z, such that $z^4 = s$. Put $y = \dfrac{z}{k^2!}$. There are, at least, two elements among the elements $my, m = \overline{0,k}$, which belong to the same class (Dirichlet principle)

$$\exists\, n, m \in N : 0 \le m < n \le k \wedge ny \sim my .$$

According to lemma 2.1 we obtain $n^2y^2 - m^2y^2 = \left(n^2 - m^2\right)y^2 \sim 0$.

Since $k \ge n > m \ge 0$, then $k^2 \ge n^2 \ge n^2 - m^2$. Hence $k^2!$ is divisible by $\left(n^2 - m^2\right)$, that is why $r = \dfrac{\left(k^2!\right)^4}{\left(n^2 - m^2\right)^2}$ is integer.

It follows from the lemma 2.3 that

$$r\left(n^2 - m^2\right)^2 y^4 = \frac{\left(k^2!\right)^4}{\left(n^2 - m^2\right)^2}\left(n^2 - m^2\right)^2 \frac{z^4}{\left(k^2!\right)^4} = z^4 = s \sim 0 .$$

Since s is arbitrary number, then all positive numbers are equivalent to zero.

If $s < 0$, then there exists $z > 0$ such that $s = -z^2$.

We proved that all positive numbers are equivalent to zero. Applying (3), we obtain $0 \sim 0 \wedge z \sim 0 \Rightarrow 0^2 - z^2 = s \sim 0^2 - 0^2 = 0$. It means that all negative numbers are also equivalent to zero. This implies that there exists only one equivalence class. But, according to the definition, there are at least two equivalence classes. The obtained contradiction **proves the theorem**.

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The introduced concept of the chromatic number of the function of two variables can be generalized to the functions of any number of the variables and to the case of the arbitrary mappings of the Cartesian power of the set into itself.

Conclusions

The concepts of the chromatic class and the chromatic number of the function have been introduced in the article. The relation between these concepts and the group theory has been obtained. It has been proved that the linear function of two real variables $f(x,y)=\alpha x+\beta y+\gamma, \alpha\beta\neq 0$ has no finite chromatic number. The similar result has been proved for the function $g(x,y)=x^2-y^2$ of the real variables.

This result can be formulated as follows:

- the functions $f(x,y)$ and $g(x,y)$ (and the function $ax_1x_2...x_n+b$ where $n>1, ab\neq 0$, which was considered in the article [1] before) can not be R - functions at any selection of the accompanying functions of the polyvalent logic;

- the set R can not be painted in the finite number of the colors in such a way that the color of the function $\alpha x+\beta y+\gamma$, where $\alpha\beta\neq 0$ could be determined by the colors of its arguments uniquely. It is also true for the functions x^2-y^2 and $ax_1x_2...x_n+b$, where $n>1, ab\neq 0$.

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